A Turing Machine is an accepting device which accepts the languages (recursively enumerable set) generated by type 0 grammars. It was invented in 1936 by Alan Turing.

Definition

A Turing Machine (TM) is a mathematical model which consists of an infinite length tape divided into cells on which input is given. It consists of a head which reads the input tape. A state register stores the state of the Turing machine. After reading an input symbol, it is replaced with another symbol, its internal state is changed, and it moves from one cell to the right or left. If the TM reaches the final state, the input string is accepted, otherwise rejected.

A TM can be formally described as a 7-tuple (Q, X, ∑, δ, q0, B, F) where −

* **Q** is a finite set of states
* **X** is the tape alphabet
* **∑** is the input alphabet
* **δ** is a transition function; δ : Q × X → Q × X × {Left\_shift, Right\_shift}.
* **q0** is the initial state
* **B** is the blank symbol
* **F** is the set of final states

Comparison with the previous automaton

The following table shows a comparison of how a Turing machine differs from Finite Automaton and Pushdown Automaton.

|  |  |  |
| --- | --- | --- |
| **Machine** | **Stack Data Structure** | **Deterministic?** |
| Finite Automaton | N.A | Yes |
| Pushdown Automaton | Last In First Out(LIFO) | No |
| Turing Machine | Infinite tape | Yes |

Example of Turing machine

Turing machine M = (Q, X, ∑, δ, q0, B, F) with

* Q = {q0, q1, q2, qf}
* X = {a, b}
* ∑ = {1}
* q0 = {q0}
* B = blank symbol
* F = {qf }

δ is given by −

|  |  |  |  |
| --- | --- | --- | --- |
| **Tape alphabet symbol** | **Present State ‘q0’** | **Present State ‘q1’** | **Present State ‘q2’** |
| a | 1Rq1 | 1Lq0 | 1Lqf |
| b | 1Lq2 | 1Rq1 | 1Rqf |

Here the transition 1Rq1 implies that the write symbol is 1, the tape moves right, and the next state is q1. Similarly, the transition 1Lq2 implies that the write symbol is 1, the tape moves left, and the next state is q2.

Time and Space Complexity of a Turing Machine

For a Turing machine, the time complexity refers to the measure of the number of times the tape moves when the machine is initialized for some input symbols and the space complexity is the number of cells of the tape written.

Time complexity all reasonable functions −

**T(n) = O(n log n)**

TM's space complexity −

**S(n) = O(n)**